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**3 (Sem-1/CBCS) MAT HC 1**

**2021**

**(Held in 2022)**

**MATHEMATICS**

(Honours)

Paper : MAT-HC-1016

**(Calculus)**

Full Marks : 60

Time : Three hours

***The figures in the margin indicate full marks for the questions.***

1. Answer the following questions :  $1 \times 7 = 7$ 
  - (a) Write down the  $n$ th derivative of  $y = \log x$ .
  - (b) The point  $P(c, f(c))$  on the graph of  $f(x)$  is such that  $f''(c) = 0$ . Does it necessarily imply that  $P$  is an inflection point on the graph ?

Contd.



(c) Write down the value of  $\lim_{x \rightarrow +\infty} x \sin \frac{1}{x}$ .

(d) Find the domain of the vector function

$$\vec{F}(t) = (1-t)\hat{i} + \sqrt{-t}\hat{j} + \frac{1}{t-2}\hat{k}$$

(e) Write one basic difference between the disk/washer and shell method for computing volume of revolution.

(f) What is the direction of velocity of a moving object on its trajectory.

(g) The velocity of a particle moving in space is  $\vec{v}(t) = e^t \hat{i} + t^2 \hat{j}$ . Find the direction of motion at time  $t=2$ .

2. Answer the following questions :  $2 \times 4 = 8$

(a) Applying L.Hopital's rule, evaluate

$$\lim_{x \rightarrow \pi/4} (1 - \tan x) \cdot \sec 2x$$

(b) Write down the parametric equation of a line that contains the point (3,1,4) and is parallel to the vector  $\vec{v} = -\hat{i} + \hat{j} - 2\hat{k}$ .

(c) Find the area of the surface generated by revolving the portion of the curve  $y = x^3$  between  $x=0$  and  $x=1$  about the  $x$ -axis.

(d) Explain briefly why the acceleration of an object moving with constant speed is always orthogonal to the direction of motion.

3. Answer **any three** of the following questions :

$5 \times 3 = 15$

(a) If  $y = \cos(m \sin^{-1} x)$ , show that

$$(1-x^2)y_{n+2} - (2n+1)x y_{n+1} + (m^2 - n^2)y_n = 0.$$

Hence find  $y_n(0)$ .

$3+2=5$

(b) Sketch the graph of a function  $f$  with all the following properties :  $5$

(i) the graph has  $y=1$  and  $x=3$  as asymptotes

(ii)  $f$  is increasing for  $x < 3$  and  $3 < x < 5$  and decreasing elsewhere

(iii) the graph is concave up for  $x < 3$  and concave down for  $3 < x < 7$

(iv)  $f(0) = 4 = f(5)$  and  $f(7) = 2$



(c) Sketch the graph of  $y = \frac{3x-5}{x-2}$  identifying the locations of intercepts, concavity and inflection points (if any) and asymptotes. 5

(d) Obtain the reduction formula for  $\int \tan^n x \, dx$ .

Hence evaluate  $\int_0^{\pi/4} \tan^5 x \, dx$  3+2=5

(e) The position vector of a moving object at any time  $t$  is given by  $\vec{R}(t) = t\hat{i} + e^t\hat{j}$ . Find the tangential and normal components of the object's acceleration. 5

4. Answer **any three** of the following questions : 10×3=30

(a) A firm determines that  $x$  units of its product can be sold daily at rupees  $p$  per unit where  $x = 1000 - p$ . The cost of producing  $x$  units per day is

$C(x) = 3000 + 20x$ . Then —

(i) Find the revenue function  $R(x)$ . 2

(ii) Find the profit function  $p(x)$ . 2

(iii) Assuming that production capacity is at most 500 units per day, determine how many units the company must produce and sell each day to maximize profit. 3

(iv) Find the maximum profit. 2

(v) What price per unit must be charged to obtain maximum profit? 1

(b) When is an object said to move in central force field? Derive Kepler's 2nd law of motion, assuming that planetary motion occurs in central force field. 2+8=10

(c) (i) Find the length of the arc of the astroid  $x^{2/3} + y^{2/3} = 1$  lying in the positive quadrant. 3

(ii) Using cylindrical shell method, find the volume of the solid formed by revolving the region bounded by the parabola  $y = 1 - x^2$ , the  $y$ -axis, and the positive  $x$ -axis, about  $y$ -axis. 4



- (iii) Find the surface area generated when the polar curve

$$r = 5, 0 \leq \theta \leq \pi/3$$

is revolved about  $x$ -axis. 3

- (d) (i) Find the volume generated by disk/washer method, when the region bounded by  $y = x$ ,  $y = 2x$  and  $y = 1$  is revolved about the  $x$ -axis 5

- (ii) A particle moves along the polar path  $(r, \theta)$  where

$$r(t) = 3 + 2 \sin t, \theta(t) = t^3.$$

Find the velocity  $\vec{v}(t)$  and acceleration  $\vec{A}(t)$  in terms  $\hat{u}_r$  and  $\hat{u}_\theta$ . 5

- (e) (i) Evaluate  $\lim_{x \rightarrow 0} (1 + \sin x)^{1/x}$ . 3

- (ii) Examine the existence of vertical tangent and cusp of the graph of  $y = (x - 4)^{2/3}$ . 3

- (iii) A projectile is fired from ground level at an angle of  $30^\circ$  with muzzle speed  $110 \text{ ft/sec}$ . Find the time of flight and the range. 4

- (f) (i) Obtain the reduction formula for  $\int \cos^n x \, dx$ .

Hence evaluate  $\int \cos^5 x \, dx$ .

$$3+2=5$$

- (ii) Find the unit tangent vector  $\vec{T}(t)$  and principal unit normal vector  $\vec{N}(t)$  at each point on the graph of vector function

$$\vec{R}(t) = (3 \sin t, 4t, 3 \cos t) \quad 5$$