Total number of printed pages–7

3 (Sem–1/CBCS) MAT HC 1

2020

(Held in 2021)

MATHEMATICS

(Honours)

Paper : MAT–HC–1016

(Calculus)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×7=7

(a) Write down the value of

$$\underset{x \to +\infty}{Lt}\ e^{-x} \cos x .$$

(b) When the line $x = c$ is a vertical asymptote of the graph of a function $f(x)$?

Contd.

(c) Define scalar triple product of three vectors $\vec{u}, \vec{v}, \vec{w}$.

(d) When Washer Method is used to compute a volume of revolution?

(e) Determine the values of t for which the function $\vec{G}(t) = t\hat{i} - \frac{1}{t}\hat{j} + \frac{1}{t-1}\hat{k}$ is continuous.

(f) Under what condition the graph of a vector function $\vec{F}(t)$ is smooth?

(g) For a production process, $C(x)$ denotes total production cost of x units and $R(x)$ denotes total revenue derived from the sale of that x units. Under what condition profit will be maximum?

2. Answer the following questions : 2×4=8

(a) Examine if $f(x) = x^4$ has a point of inflection at $x = 0$.

(b) Find the nth derivative of $y = xe^{ax}$, using Leibniz's rule.

(c) Obtain the reduction formula for $\int \tan^n x \, dx$.

(d) Evaluate $\int_0^{\pi} \left(t\hat{i} + 3\hat{j} - \sin t\hat{k}\right) dt$.

3. Answer **any three** of the following questions :

(a) If $y = \sin\left(m \sin^{-1} x\right)$, show that 2+3=5

(i) $\left(1 - x^2\right) y_2 - x y_1 + m^2 y = 0$

(ii) $\left(1 - x^2\right) y_{n+2} = (2n+1) x y_{n+1} + \left(n^2 - m^2\right) y_n$

(b) Sketch the graph of the following ***(any one)*** equation. 5

(i) $y = 4 + \frac{2x}{x-3}$

(ii) $g(t) = \left(t^3 + t\right)^2$

identifying the locations of intercepts, inflection points *(if any)* and asymptotes.

(c) Obtain the reduction formula for $\int \sin^m x \cos^n x \, dx$. 5

(d) Using Washer method find the volume of the solid formed by revolving the region bounded by $x = y^2$ and $y = x^2$ about *(i)* x-axis, *(ii)* y-axis. 5

(e) The position vector of a moving body is $\vec{R}(t) = 2t\hat{i} - t^2\hat{j}$ for $t \geq 0$. Express $\vec{R}$ and velocity vector $\vec{v}(t)$ in terms $\hat{u}_r$ and $\hat{u}_\theta$, $\hat{u}_r$ and $\hat{u}_\theta$ being unit vectors along and perpendicular to radial axis. 5

Answer ***any three*** *of the following questions :*

4. (a) Evaluate the following using L' Hôpital's rule

   (i) $\underset{x \to 0}{Lt} \dfrac{\log x}{\operatorname{cosec} x}$ 2

   (ii) $\underset{x \to +\infty}{Lt} \left(1 + \dfrac{1}{2x}\right)^{3x}$ 3

   (b) A liquid form of antibiotic manufactured by a pharmaceutical firm is sold in bulk at a price of Rs. 200 per unit. If the total production cost (in Rs.) for x units is $C(x) = 5{,}00{,}000 + 80x + {\cdot}003x^2$ and if the production capacity of the firm is atmost 30,000 units in a specified time, how many units of antibiotic must be manufactured and sold in that time to maximize the profit? 5

5. (a) Find the length of the curve defined by $9x^2 = 4y^3$ between the points $(0, 0)$ and $(2\sqrt{3}, 3)$. 3

   (b) Find the length of the polar curve $r = \cos\theta$. 3

   (c) Using cylindrical shell method find the volume of the solid formed by revolving the region bounded by the lines $y = 2x$, the y-axis and $y = 1$ about y-axis. 4

6. (a) Given $\vec{F}(t) = \hat{i} + t\hat{j} + t^2\hat{k}$ and $\vec{G}(t) = t\hat{i} - e^t\hat{j} + 3\hat{k}$. Find

   $\dfrac{d}{dt}\left[\vec{F}(t) \times \vec{G}(t)\right]$. 3

   (b) Find a vector function $\vec{F}$ whose graph is the curve of intersection of the hemisphere $z = \sqrt{4 - x^2 - y^2}$ and the parabolic cylinder $y = x^2$. 2

(c) A projectile travels in vacuum in a co-ordinate plane, with x-axis along the level ground. If the projectile is fired from a height of s_0 with initial speed v_0 and angle of elevation α, then prove that at time t $(t \geq 0)$ it will be at the point $(x(t), y(t))$ where

$x(t) = v_0 \cos\alpha . t$ and

$y(t) = v_0 \sin\alpha . t - \frac{1}{2} g t^2 + s_0$. 5

7. (a) Examine if $f(x) = x^{1/3}(x-4)$ has a vertical tangent at $x = 0$. 2

(b) If a non-zero vector function $\vec{F}(t)$ is differentiable and has constant length, then $\vec{F}(t)$ is orthogonal to the derivative vector $\vec{F}'(t)$ — Prove it. 3

(c) An object moving along a smooth curve (with $T' \neq 0$) has velocity $\vec{V}$ given by $\vec{V} = \frac{ds}{dt}\hat{T}$.

Deduce the expression for acceleration in the form

$$\vec{A} = \frac{d^2s}{dt^2}\hat{T} + k\left(\frac{ds}{dt}\right)^2 \hat{N}$$

(Symbols having their usual meanings.) 5

8. (a) A manufacturer estimates that when x units of a particular commodity are produced each month, the total cost (in Rs.) will be

$$C(x) = \frac{1}{8}x^2 + 4x + 200$$

and all units can be sold at a price of

$p(x) = 49 - x$ rupees per unit.

Find average cost, marginal cost and marginal revenue for this production process. 2+2+2=6

(b) Derive the formula for surface area of a sphere of radius r. 4