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3 (Sem-1/CBCS) PHY HC I

2020

(Held in 2021)

PHYSICS

(Honours)

Paper : PHY-HC-1016

(Mathematical Physics-I)

Full Marks : 60

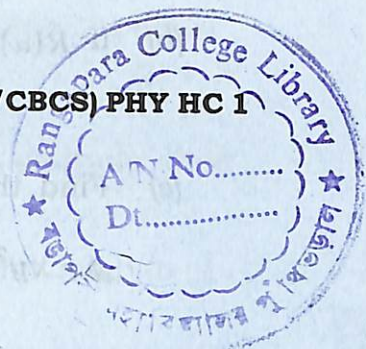
Time : Three hours

**The figures in the margin indicate
full marks for the questions.**

1. Answer the following questions : $1 \times 7 = 7$

- (a) What is the geometrical interpretation of the scalar triple product of three vectors ?

Contd.



(b) If $\bar{R}(u) = \frac{d}{du} \bar{S}(u)$, find $\int_a^b \bar{R}(u) du$.

(c) Find the Laplacian of the scalar field

$$\phi = xyz^2z^3.$$

(d) Determine the order and degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right) + x^2\left(\frac{dy}{dx}\right)^2 = 0$$

(e) What are the coordinate surfaces in orthogonal curvilinear coordinates?

(f) Define Dirac delta function.

(g) Write the difference between Systematic error and Random error.

2. Answer **any four** of the following questions :

2×4=8

(a) If $\bar{A}(t)$ has a constant magnitude, then

show that $\frac{d\bar{A}}{dt}$ is perpendicular to \bar{A} .

(b) Prove that, the vector

$$\bar{A} = 3y^4z^2\hat{i} + 4x^3z^2\hat{j} - 3x^2y^2\hat{k}$$
 is solenoidal.

(c) Show that $\iint_S \bar{A} \cdot \hat{n} dS$, over any closed

$$\text{surface } S \text{ is equal to } \iint_R \bar{A} \cdot \hat{n} \frac{dx dy}{|\hat{n} \cdot \hat{k}|},$$

where R is the projection of S on xy -plane.

(d) Solve the differential equation

$$xy(y+1)dy = (x^2+1)dx.$$

(e) State the transformation relation between the spherical polar coordinates (r, θ, ϕ) and Cartesian coordinates (x, y, z) . Obtain the volume elements in spherical polar co-ordinate.

3. Answer **any three** of the following questions : 5×3=15

(a) How will you define divergence and curl of a vector \vec{v} . Evaluate $\vec{\nabla} \cdot \vec{r}$ and $\vec{\nabla} \times \vec{r}$.

(b) If \vec{A} is a vector, prove that $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla} (\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A}$.

(c) Test the Exactness of the differential equation $(5x^4 + 3x^2y^2 - 2xy^3) dx + (2x^3y - 3x^2y^2 - 5y^4) dy = 0$ and then solve it.

(d) Express $\nabla^2 \psi$ in orthogonal curvilinear coordinates.

4. Answer **any three** of the following questions : 10×3=30

(a) (i) Show that the surface integral of a vector \vec{F} and the volume integral of the divergence of the same vector obey the relation :

$$\iint_S \vec{F} \cdot d\vec{S} = \iiint_V (\vec{\nabla} \cdot \vec{F}) dV \quad 6$$

(ii) Evaluate $\iint_S \vec{r} \cdot \hat{n} dS$, where S is a closed surface. 4

OR

(b) Prove that $\oint_C \vec{A} \cdot d\vec{\lambda} = \int_S (\vec{\nabla} \times \vec{A}) \cdot d\vec{S}$,

where C is the curve bounding the surface S . Hence find $\oint \vec{r} \cdot d\vec{r}$.

8+2=10

(c) Solve the following differential equations: 5+5=10

(i) $(1+x^2)\frac{dy}{dx} + 2xy = \cos x$

(ii) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$, subject to the condition $y(0)=0$ and $y'(0)=1$.

(d) (i) Prove that spherical polar coordinate system is orthogonal. 6

(ii) The probability density function of a variable X is

$X:$	0	1	2	3	4	5	6
$P(X):$	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

Find $P(X < 4)$, $P(X \geq 5)$, $P(3 < X \leq 6)$. Here $P(X)$ is a probability density function. 4

(e) (i) Prove the expression

$$\int_{-\infty}^{+\infty} \delta(x) dx = 1 \text{ where } \delta(x) = 0 \text{ if}$$

$$x \neq 0 \text{ and } \delta(x) = \infty \text{ if } x = 0. \quad 5$$

(ii) Given the three vectors

$$\vec{A} = \hat{i} + 2\hat{j} - \hat{k}$$

$$\vec{B} = \hat{j} + \hat{k}$$

$$\vec{C} = \hat{i} - \hat{j}$$

Evaluate $\vec{A} \times (\vec{B} \times \vec{C})$ and show that

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$$

2+3=5