Total number of printed pages–7

3 (Sem–1/CBCS) PHY HC 1

2020

(Held in 2021)

PHYSICS

(Honours)

Paper : PHY–HC–1016

(Mathematical Physics–I)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×7=7

   (a) What is the geometrical interpretation of the scalar triple product of three vectors ?

(b) If $\vec{R}(u) = \frac{d}{du}\vec{S}(u)$, find $\int_a^b \vec{R}(u)\, du$.

(c) Find the Laplacian of the scaler field $\phi = xy^2z^3$.

(d) Determine the order and degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right) + x^2\left(\frac{dy}{dx}\right)^2 = 0$$

(e) What are the coordinate surfaces in orthogonal curvilinear coordinates?

(f) Define Dirac delta function.

(g) Write the difference between Systematic error and Random error.

2. Answer ***any four*** of the following questions : 2×4=8

(a) If $\vec{A}(t)$ has a constant magnitude, then show that $\frac{d\vec{A}}{dt}$ is perpendicular to $\vec{A}$.

(b) Prove that, the vector $\vec{A} = 3y^4z^2\hat{i} + 4x^3z^2\hat{j} - 3x^2y^2\hat{k}$ is solenoidal.

(c) Show that $\iint_S \vec{A}.\hat{n}\, dS$, over any closed surface S is equal to $\iint_R \vec{A}.\hat{n}\,\frac{dx\,dy}{|\hat{n}.\hat{k}|}$, where R is the projection of S on xy-plane.

(d) Solve the differential equation

$$xy\,(y+1)\,dy = \left(x^2+1\right)dx.$$

(e) State the transformation relation between the spherical polar co-ordinates (r, θ, ϕ) and Cartesian co-ordinates (x, y, z). Obtain the volume elements in spherical polar co-ordinate.

3. Answer ***any three*** of the following questions : 5×3=15

(a) How will you define divergence and curl of a vector $\vec{V}$. Evaluate $\vec{\nabla}.\vec{r}$ and $\vec{\nabla}\times\vec{r}$.

(b) If $\vec{A}$ is a vector, prove that $\vec{\nabla}\times\left(\vec{\nabla}\times\vec{A}\right)=\vec{\nabla}\left(\vec{\nabla}.\vec{A}\right)-\nabla^2\vec{A}$.

(c) Test the Exactness of the differential equation

$$\left(5x^4+3x^2y^2-2xy^3\right)dx+\left(2x^3y-3x^2y^2-5y^4\right)dy=0$$

and then solve it.

(d) Express $\nabla^2\psi$ in orthogonal curvilinear coordinates.

4. Answer ***any three*** of the following questions : 10×3=30

(a) (i) Show that the surface integral of a vector $\vec{F}$ and the volume integral of the divergence of the same vector obey the relation :

$$\iint_S \vec{F}.d\vec{S}=\iiint_V\left(\vec{\nabla}.\vec{F}\right)dV$$ 6

(ii) Evaluate $\iint_S \vec{r}.\hat{n}\,dS$, where S is a closed surface. 4

OR

(b) Prove that $\oint_C \vec{A}.d\vec{\lambda}=\int_S\left(\vec{\nabla}\times\vec{A}\right).d\vec{S}$, where C is the curve bounding the surface S. Hence find $\oint \vec{r}.d\vec{r}$. 8+2=10

(c) Solve the following differential equations : 5+5=10

(i) $\left(1+x^2\right)\dfrac{dy}{dx}+2xy=\cos x$

(ii) $\dfrac{d^2y}{dx^2}-3\dfrac{dy}{dx}+2y=0$, subject to the condition $y(0)=0$ and $y'(0)=1$.

(d) (i) Prove that spherical polar coordinate system is orthogonal. 6

(ii) The probability density function of a variable X is

X:	0	1	2	3	4	5	6
$P(X)$:	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

Find $P(X<4)$, $P(X\geq 5)$, $P(3<X\leq 6)$. Here $P(X)$ is a probability density function. 4

(e) (i) Prove the expression

$\int_{-\infty}^{+\infty}\delta(x)dx=1$ where $\delta(x)=0$ if $x\neq 0$ and $\delta(x)=\infty$ if $x=0$. 5

(ii) Given the three vectors

$$\vec{A}=\hat{i}+2\hat{j}-\hat{k}$$
$$\vec{B}=\hat{j}+\hat{k}$$
$$\vec{C}=\hat{i}-\hat{j}$$

Evaluate $\vec{A}\times\left(\vec{B}\times\vec{C}\right)$ and show that

$$\vec{A}\times\left(\vec{B}\times\vec{C}\right)=\vec{B}\left(\vec{A}\cdot\vec{C}\right)-\vec{C}\left(\vec{A}\cdot\vec{B}\right)$$

2+3=5

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